In Search of Simulating Reality: Validation and Updating of FE Models for Structural Analysis

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Summary

To successfully make the move to digital prototyping, and thereby reduce the number of physical prototypes, predictions of product performance should be provided with a measure of confidence and validated against experimental data.
Typical applications are found in the field of modal analysis where frameworks and efficient procedures are now available for calibrating analytical models to better correspond with the results of experimental models .
An overview of the methodology is provided along with some examples in material identification and FE model refinement.

Keywords

simulation, validation, correlation, model updating, optimization, material identificaiton

0. Introduction

To successfully make the move to digital prototyping, and thereby reduce the number of physical prototypes, predictions of product performance should be provided with a measure of confidence and validated against experimental data. This will ultimately require qualifying and quantifying the physical and numerical uncertainty on the simulation model.

In a deterministic approach, model validation and updating is based on nominal values for input parameters and average test results.

Probabilistic analysis offers a way to study the propagation of uncertainty throughout the structure. Scatter information on simulated structural responses is used to compare with point clouds that are the result from repeated testing. This offers a more complete basic for checking the validity of a model and setting realistic target for model updating [1].

1. Uncertainty in Simulation

Uncertainty in numerical simulation results manifests itself in two main classes: physical uncertainty and numerical uncertainty.

There exist four main levels at which physical uncertainty, or scatter, becomes visible, namely

- **Boundary and initial conditions** - impact velocity, impact angle, mass of vehicle, characteristics of barriers, etc.
- **Material properties** - yield stress, strain-rate parameters, density, local imperfections, etc
- **Geometry** - shape, thickness, manufacturing and assembly tolerances, etc.
- **Loads** - earthquakes, wind gusts, sea waves, blasts, shocks, impacts, etc.

Uncertainty is further increased because many of these properties may vary substantially with temperature, frequency, or load level. Information on these forms of scatter can be obtained by measurement. A sufficient large number of samples need to be evaluated to distinguish the natural and intrinsic scatter from the (often high) scatter that may be attributed to a small number of statistical samples.

Probability distribution functions and their associated properties can be obtained from statistical analysis of the test data. For example, the elastic modulus of isotropic material can be described using a normal (Gaussian) distribution that is characterized by a mean value and standard deviation.

The following types of numerical uncertainty can be identified:

- **Conceptual modeling uncertainty** - lack of data on the physical process involved, lack of system knowledge.
- **Mathematical modeling uncertainty** - accuracy of the mathematical model validity.
- **Discretization error uncertainties** – the choice of element types, mesh density, level of geometrical detail.
- **Numerical solution uncertainty** - rounding-off, convergence tolerances, integration step.
- **Human mistakes** - programming errors in the code or wrong utilization of the software, mistakes in data or units.

These types of scatter may or may not exist regardless of the physics involved. An example of the exhibit of numerical uncertainty is the different results that may be obtained by two finite element codes, using the same finite element model. Indeed changing solver, computing platform, or element formulation can be possible causes of significant differences.

It is clear that uncertainty also exists in testing. Possible causes of physical uncertainty are related to:

- **Test definition** – fixture, mounting procedure, excitation method, transducer location, sensor weight, dynamic loading.
- **Instrumentation** – calibration, distortions, cabling noise.
- **Data acquisition** – digital signal processing, measurement and filtering error.

Techniques like experimental modal analysis, are also subject to numerical uncertainty in the mathematical models that are used for modal parameter estimation.

Recognizing the existence of uncertainty and scatter is a sufficient reason to spend more time on validating simulation models. However, validation of a single model using a single test leads to only a snapshot result. While this may be valuable for roughly calibrating the input parameters, it should be understood that information on the broader picture is not taken into account unless scatter on the input parameters and reference test data is taken into account. Depending on the purpose of the simulation, and the amount and type of reference test data that is available, different situations can be encountered:

- Selection of parameters based on an hierarchy defined by uncertainty and sensitivity level (joints, materials,...). This can require different actions like equivalent geometrical parameter updating, material identification, mesh refinement or load identification.
- The mesh itself determines the updating parameters. Some examples are stick models for wing flutter analysis , component models that need to be reduced and included in an assembly (synthesis) or just for computational efficiency, refined models to be used for both dynamics and static analysis (stress, optimization), models to be used for acoustic analysis, etc.
- Scatter on output responses, obtained from repeated testing of the same structure or testing many samples, can be used to identify or adjust estimates of random properties of input parameters leading to a validated and updated stochastic simulation model.

From the previous paragraphs it should be clear that developing the proper model validation and updating strategy is often a complex issue and should be looked at as a process that must be adapted to specific needs.

A survey among finite element users showed that highest concern is expressed about using an appropriately refined mesh and having access to correct material properties [2]. For this reason it is shown hereafter how these can be validated and improved using a deterministic or propabilistic model updating methodology.

2. Deterministic Model Updating

Most correlation and updating techniques for structural dynamics applications are based on the functional relationship between the measured responses and the structural parameters that can be expressed in terms of a Taylor series expansion limited to the linear term [3]. This relationship can be written as:

$$\{R_e\} = \{R_a\} + [S](\{P_u\} - \{P_o\}) \qquad (1)$$

Where:

$\{R_e\}$ Vector containing the reference system responses (experimental data).
$\{R_a\}$ Vector containing the predicted system responses for a given state $\{P_o\}$ of the parameter values.
$\{P_u\}$ Vector containing the updated parameter values.
$[S]$ Sensitivity matrix.

The discrepancy between the initial model predictions and the test data is resolved by minimizing a weighted error E, given by:

$$Min(E = \{\Delta R\}^t [C_R]\{\Delta R\} + \{\Delta P\}^t [C_P]\{\Delta P\}) \qquad (2)$$

and subject to constraints

$$g_i(P) \leq 0 \quad ; P_{\min} \leq P \leq P_{\max} \tag{3}$$

The matrices $[C_R]$ and $[C_P]$ respectively express the confidence of the user in the reference system responses and initial parameter estimates. In case statistical data is available, the covariance matrices can be used weigh the responses and parameter values.

Minimizing E with respect to the parameter values, leads to a updated value for the parameter values that reduce the distance between the simulation and test results, i.e. reducing the not only the residues between reference and predicted residues but at the same time keep the change between the original and updated model minimal (in terms of parameter changes).

The similarity of model updating defined by equations (2) and (3) with a general design optimization problem is striking. However, whereas the 'objective function' in design optimization usually expresses the quality of the design in terms of cost, weight, reliability, etc., model updating is concerned about improving the finite element model to better predict the observed behavior while at the same time limiting the changes to the model. This translates to different choices of targets, variables and constraints. In design optimization, an optimum is acceptable if the parameter constraints are satisfied. In model updating, the changes to the model should remain within the range of expected variance of the input parameters. This is not only guaranteed by satisfying the proper parameter constraints but also by updating the most suitable parameters.

Another major issue is that the system of equations (1) is usually underdetermined i.e. the number of updating parameters largely exceeds the number of responses. Possible solutions are to reduce the number of updating parameters or increase the number of responses. Some ways to reduce the number of updating parameters are:

- Using techniques like local correlation analysis, sensitivity analysis and uncertainty analysis to reduce the model size in terms of number of critical combinations of input parameters.
- Grouping elements and selecting global parameters for the group instead of updating at local element level.
- Defining many parameter relations (equation (3)) and thus reducing the number of independent parameters. It is to be noted that the $[C_P]$ matrix in equation (2) also offers the potential to express relations between parameters.
- Using a bottom-up modeling and test methodology. In this approach the different components that constitute an assembly are first modeled, tested and updated separately. This is followed by the repeated tests at different phases of the assembly that allows focusing on modeling of joints.

On the other hand, the number of responses can be increased by:

- Adding correlation targets. Not only responses can be included that are directly measurable like mass, displacements, frequency response functions or resonance frequencies, but also computed correlation targets like modal assurance correlation [4], or mode shape orthogonality (figure 1).
- Simultaneously updating the parameters that are common in variants of the FE model. For example, solar panels for satellites can be tested during different stages of deployment and for each stage there is a FE model. This provides a richer set of test data to serve as references for updating element properties that are common in all configurations. Such properties can be, for example, the joint stiffness or material properties. Other examples are a launcher tested with different levels of fuel, or differently shaped test specimens made of a composite material that needs to be identified [5].
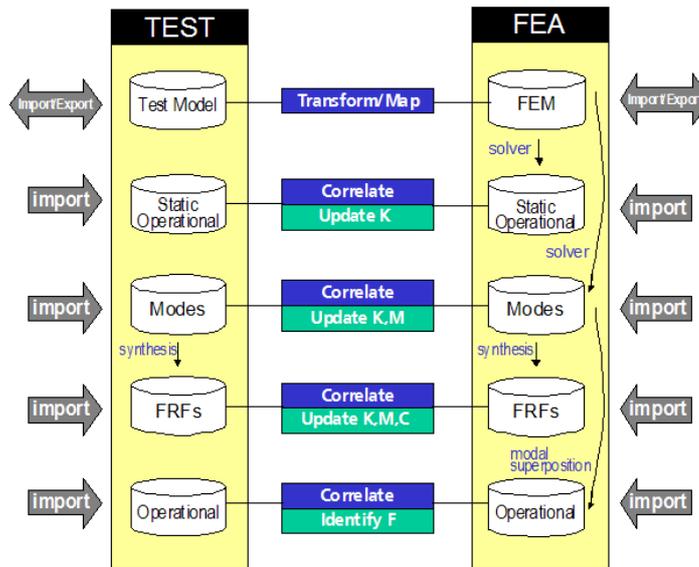- Using full field measurement data (laser, optical).

Figure 1: Test-analysis correlation and FE model updating for structural analysis can involve static or dynamic response data to update mass (M), stiffness (K), damping (C) or loads (F).

3.   Probabilistic Model Validation and Updating

In the presence of scatter, the single deterministic response represents only one point of a cloud of values and therefore carries little information on likeliness and trends. Point clouds on the other hand can be interpreted in term of probability of a response value lying below or above a prescribed level. At a minimum, all responses are now defined as an interval with information on the confidence an analyst can have that the true response value will be within this interval. Additional statistical information can be derived if necessary.

Obtaining test point clouds requires repeated testing of a same product (to reveal physical scatter) as well as testing a series of similar products to identify product variability. From the simulation side, the updating parameters can be randomized (i.e. apply a statistical probability distribution) and, using a probabilistic analysis tool like Monte Carlo Simulation or other, obtain point clouds for every parameter- response combination. There is a point for each state (also called sample) of the input variable. On statistical grounds, the collection of all point clouds, one for each input-output variable combination, constitute a new concept of model, often referred to as Meta-Model in literature. This model can be complemented by input-input and output-output relationships. From these point clouds, statistical postprocessing results like histograms, the mean and standard deviation of output responses are obtained.

The correlation between simulated and experimentally obtained point clouds should now be analyzed using statistical measures. An example is the Mahalonobis distance:

$$d_M = (\mu_1 - \mu_2)^t \, COV_p^{-1} \, (\mu_1 - \mu_2) \qquad (4)$$

where the vectors μ_1 and μ_2 represent the centers of gravity of each meta model and COV_p the pooled covariance matrix. Note the similarity with equation (3). Whereas a deterministic measure of correlation, like the average relative error on resonance frequencies, provides only a snapshot measure that could be good or bad depending on coincidence, the Mahalonobis distance is clearly a much safer measure because it is based on position and shape of point clouds. Coincidence, good or bad luck with parameter estimations or variable measurement conditions can hardly influence this result.

The concept of meta models, both for numerical simulation and testing, together with the Mahalonobis metric, enables comparing responses in a statistically sound and rigorous manner. Position, shape and size of point clouds should be compared with the test meta-model being the reference. For example, consider the scatter plots shown in figure 2. Differences in the principal axes of the two ellipses suggest either a major shortcoming in the discretization of structure geometry, a physical discrepancy between the two models or simply modelling errors. It should be clear that relative translation and overall size of point clouds are easier to correct than relative rotations. The former merely indicate systematic or global errors whereas the latter usually indicate (local) physical errors.

Secondly, the level of scatter in the two models is clearly different. Although this may be desirable in some cases, it is in general preferable to obtain a simulation model that exhibits a level of scatter that is in balance with the scatter on the test data.
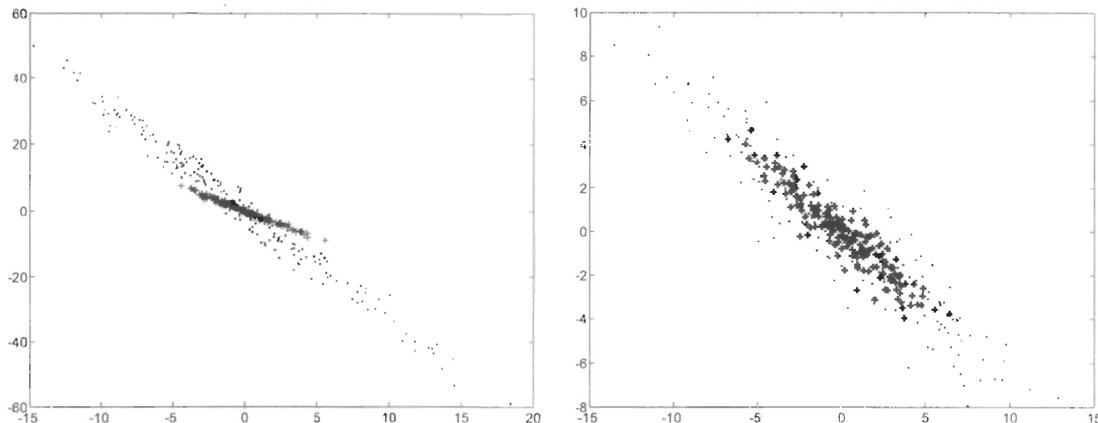

Figure 2: Example of a physically distant (left) and close (right) models represented by means of point clouds.

A fundamental contribution of meta-model analysis towards model updating is the possibility of pinpointing the dominating parameters of a system and to quantify the correlations between the input and output variables. This is the equivalent of sensitivity analysis in deterministic analysis. However, the concept of sensitivities, or gradients, no longer exists in the presence of scatter. So unless scatter is very low and can be neglected, other procedures to identify the dominant parameters need to be applied. In a similar way, not all available responses may be of equal relevance. Indeed, statistical postprocessing may reveal hidden relations and identify dependent and independent responses. As a result, the analyst can reduce the order of the system to include only the most dominant parameters and independent responses. Using regression analysis, relations between the dominant parameters and independent responses are established. This holds the promise to solve some of the remaining difficulties in deterministic model updating like selection of updating parameters, definition of targets (i.e. when is correlation satisfactory) and interpretation of results. Without the additional insight that can be gained from probabilistic analysis, these decisions have to be made mainly based on engineering judgment.

The objective of probabilistic model updating is then to solve the system of equations for unknown parameter properties that change the centre of gravity, the principal directions and the density of point clouds resulting from probabilistic analysis to match the corresponding test point clouds. In fact this comes down to 'updating' the Probability Density Function (PDF) of input parameters such that the PDF of the outputs correspond with the PDF of the experimental reference responses. In its simplest form, assuming a normal probability distribution, this means that in addition to the nominal value (like in deterministic model updating), also the standard deviation of model parameters should be adjusted.

It should be noted that the ranking of input parameters based on how much they influence the performance of the system, offers additional benefits in the subsequent design improvement phase. Indeed, a designer or engineer does not need to spend time with input parameters that have only minor influence. Instead the functional performance of the design can be modified most efficiently by working with the most dominant parameters only. Reducing the scatter on these parameters (for

example by specifying more severe manufacturing tolerances) is the most rewarding in terms of robustness of the design. On the other hand, the engineer should relax tolerances on the parameters that do not significantly influence the performance, and in the process save money on manufacturing costs.

4.   Dedicated Software for Model Validation and Updating

FE model updating is a knowledge and decision-based process that requires access to a multitude of tools for database management, response and parameter selection, and numerical tools for correlation analysis, sensitivity analysis and parameter estimation. This is best achieved in a dedicated environment with a user interface and graphics that are purposely designed for these tasks.

An example of a special purpose model updating software is FEMtools [6], which is compatible with ANSYS. A description with application examples is provided in the paper „Anwending von Model Updating mit FEMtools", from the same authors and also presented at this 2005 CADFEM User's Meeting.

5.   Examples

Two major concerns of an analyst validating an FE model are to make sure that the model includes the right features and thus is appropriate in terms of the mesh discretisation, and secondly that the input parameters (material, equivalent geometrical properties, joint stiffness,...) are correct.

5.1   Local Model Refinement

The use of multiple targets to be matched by a simulation model i.e. resonant frequencies, mode shapes, frequency response, etc. may require local changes of stiffness and mass. With sensitivity analysis and model updating as described in section 2. it is possible to visualize the areas in a model that need changes. This can be interpreted as areas where the fidelity of the model need to be reviewed. In practice this may require adding detail, remesh, or increase mesh density.

Figures 3 and 4 show the results obtained using an initially very coarse model to simulate the first seven mode shapes of a hard disk enclosure. Figure 5 shows the areas in a engine housing that require refinement. In both cases experimental modal analysis was used to obtain reference target data.
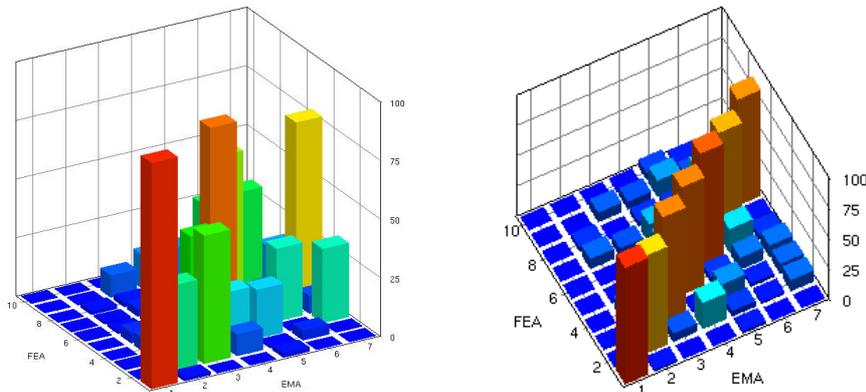

Figure 3: Modal correlation between a set of simulated mode shapes and experimentally obtained modes. Before local modification of the model only a few modes correlate well (left). After local modification all modes appear in the right order and correlate well with test (right).
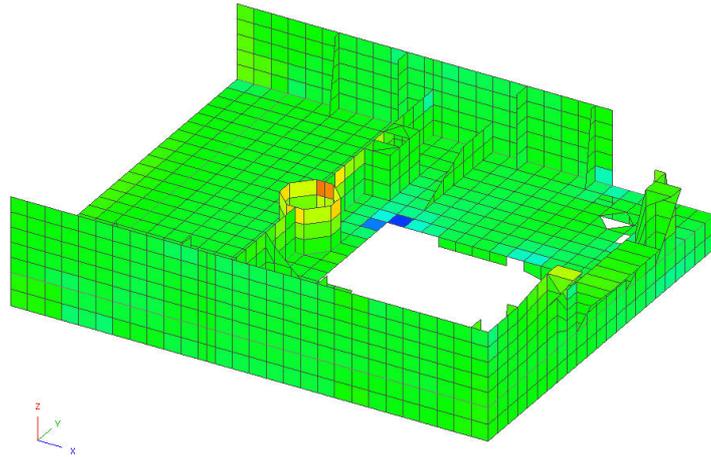
Figure 4: Areas of a coarse model that need local refinement in order to predict resonance frequencies and mode shape closer to test data (see also figure 3). Some parts need remeshing using a shape closer to the real structure (cylinder), other parts require higher mesh densitiy to correctly capture stress gradients near geometrical discontinuities.
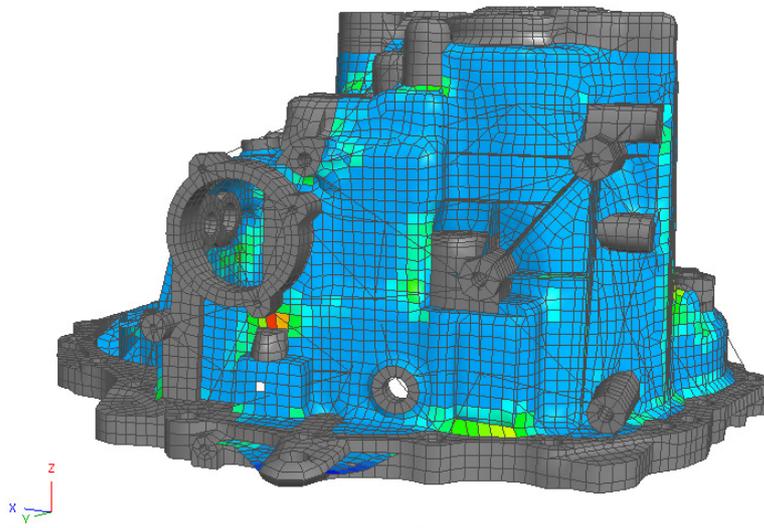

Figure 5: Areas of a engine model that need local refinement based on correlation with 10 experimentally obtained mode shapes up to 2000 Hz.

5.2   Material Identification

Using relevant material data that applies to the model at hand is critical to improve the realism of simulation. Variations due to manufacturing or operating conditions may have significant influence on the predicted structural responses and cannot be neglected. Correlation of FE simulations with test and updating of material properties has proven to be an efficient methodology to obtain material data in a non-destructive way [7-9].

The general flow chart and schematic view of an experimental setup are shown in the following figures.
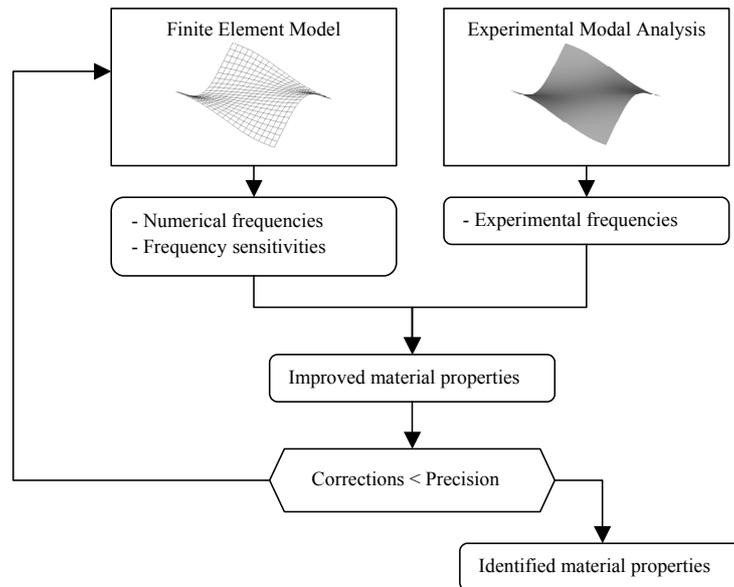
Figure 6: General flow-chart for material identification.

In recent years, this method has been demonstrated for identification of

- Slender pultruded beams with complex cross-sectional geometries.
- Laminated composites plates and sandwich materials.
- Orthotropic material directions in rolled steel plates.
- Filament-wound cylinders and pressure vessels.
- Properties of coated metallic plates (thermal barriers).
- Local variations in the Young's modulus of cast iron parts.

The method provides for fast identification using tests that represent the real material in operating conditions. Using non-contact excitation (e.g. speaker) and measurement (e.g. laser), the test setup can be easily adapted to be used in temperature-controlled environments to obtain properties as function of temperature (or other controlled conditions).
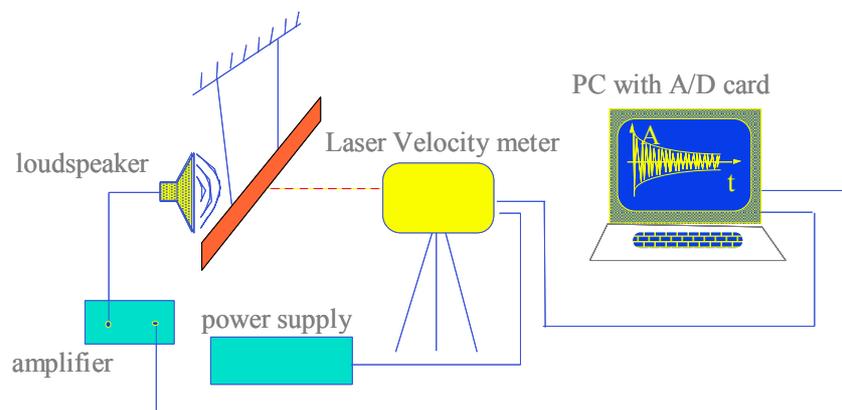

Figure 7: Schematic view of an experimental set-up for material identification using laser velocity measurement.

Conclusions

Finite element analysis has become an essential tool to support virtual product development. To successfully make the move to digital prototyping, and thereby reduce the number of physical prototypes, predictions of performance should be provided with a measure of confidence and validated against experimental data. Validation FEA is a complex process that touches all aspects of the engineering design and analysis cycle. The complexity and nature of this task requires dedicated software tools.

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